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## Constitutions as Equilibria: A Game-theoretic Approach to Positive Constitutional Economics

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# Constitutions as Equilibria: A Game-theoretic Approach to Positive Constitutional Economics

by Roland Kirstein\*  
and Stefan Voigt \*\*

Center for the Study of Law and Economics  
Discussion Paper 9904; July 8, 1999

## Abstract

The aim of this paper is to derive conditions under which either dictatorship or the rule of law are the equilibria of a post-constitutional game. It thus contributes to positive constitutional economics, i.e., the research program that is interested in explaining the emergence of constitutions and their change over time. In our model, society is assumed to consist of two groups one of which has a comparative advantage in using violence. Violence can be used to produce (transactional) security as well as to exploit the weaker group, which has a comparative advantage in producing a private good. Yet, exploitation is limited: it increases the chances of a revolution and reduces the incentives of the exploited group to produce the private good. The model identifies the conditions under which the two groups will comply with a social contract which consists of the exchange of high effort in producing the private good against provision of security. We also identify conditions under which a social contract is cheap talk and exploitation occurs.

**JEL-Classification:** *H 11, P 51, D 72*

**Encyclopedia of Law and Economics:** *0610, 0550, 9100*

**Keywords:** *Self-enforcing contracts, Rule of law, dictatorship, autocracy, Positive Constitutional Economics.*

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# 1. Introduction

Many societies are governed by autocrats. Many other governments remain within the confines of the rule of law. We observe that societies which have been governed by autocrats are able to depose of them and install a government that does not make itself better off by exploiting its citizens. But we also observe that governments who had refrained from exploitation for a long time end up as autocrats. These observations and developments all deal with de facto constitutions, i.e., with the capacities and restrictions of governments as they factually apply and not as they are written down in solemn form in the document usually called the constitution.

A comprehensive economic theory capable of explaining these changes is not available yet. Public Choice is basically interested in explaining the working properties of political systems assuming a given set of rules. Its explanatory focus is furthermore heavily tilted toward explaining the working properties of democratic systems. The economic theory of autocracy is still in its infancy<sup>1</sup>. The same holds for an economic theory<sup>2</sup>

- that identifies the conditions under which autocracy is to be expected, and those under which the rule of law will prevail,
- and that furthermore explains the transition from one form of government to the other.

It is the aim of this paper to make a step toward such a theory. To do so, we make use of non-cooperative game theory<sup>3</sup>. As in BUCHANAN (1975), we distinguish between a constitutional and a post-constitutional stage. However, we do not assume the constitutional stage to take place only once, followed by a sequence of post-constitutional interaction. We rather assume a sequence of games, each of which consists of a constitutional and a post-constitutional stage, and disregard the possibility of intertemporal decisions such as supergame strategies.

As our focus is on the non-cooperative interaction in the post-constitutional stage, we take the result of the negotiations during the constitutional stage as exogenously given. The aim of the model is to show whether the parties have an incentive to comply with the constitutional agreement. If the equilibrium in the post-constitutional stage is compatible with this social contract, then, at least in the round under consideration, it will indeed become effective. If the parties have an incentive not to stick to their promises made during the constitutional stage, then the social contract is nothing but cheap talk.

We assume society to consist of two groups of individuals, one of which has a comparative advantage in producing security, whereas the other one has a comparative advantage in producing a private good<sup>4</sup>. A social contract between these two groups consists of the

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<sup>1</sup>TULLOCK (1987) is one of the few monographs on the subject.

<sup>2</sup>VOIGT (1999) is a first sketch.

<sup>3</sup>See e.g. COOTER (forthcoming), who deals with constitutional issues drawing on game theory

<sup>4</sup>Following NORTH (1981).

promise of the weaker group to produce the private good at a high level, whereas the more violent group promises not to demand more than the agreed upon share of this output in exchange for providing security<sup>5</sup>. In a number of papers, MANCUR OLSON has dealt with the incentives of groups who have a comparative advantage in the expropriation of others<sup>6</sup>. He has shown that "roving bandits" have an incentive to become stationary and to protect the property of a given population if that increases saving and investment and will enable the stationary bandits to get a higher sum total out of that group. Olson also shows that the optimal rate of extortion - or tax - is lower still if it is determined by majority decision. In this paper, we go beyond the Olson-approach in that we model the strategic interaction between the two groups explicitly.

In this paper, the conditions under which different types of equilibria can be predicted are systematically specified<sup>7</sup>. This allows us to identify parameter changes which cause transformations from one form of government to another. An empirical evaluation of the parameter developments in order to test whether our predictions concerning constitutional change conform with empirically observable change will have to await another paper.

We proceed as follows:

- Section two delineates our positive approach toward constitutional economics from the hitherto dominating normative one.
- Section three contains our model.
- Section four concludes the paper and reiterates some of the open questions.

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<sup>5</sup>SCHMIDT-TRENZ (1996, 27) distinguishes models in which the provision of the public good may face the so called "contribution problem" or the so called "delegation problem". We focus on a situation where the contribution problem does not arise, since the more violent group simply can demand a share of the private group. Furthermore, the delegation problem is explicitly ruled out in our model, since the violent group is assumed to produce security anyway. Rather, we analyze whether the violent group demands more than the agreed upon share of the private good.

<sup>6</sup>E.g. OLSON (1991), but also MCGUIRE/OLSON (1996).

<sup>7</sup>See also GROSSMAN (forthcoming) who argues that a high survival probability of the state is a precondition for the state to credibly commit itself to non-expropriation. In other words: The higher the capability of the citizens to depose of their government, the lower the chances that government will act as if it were an agent of the citizenry.

Concerning parliament as an organization that allows autocrats to credibly bind themselves, BARZEL (1997) argues that secure kings deliberately gave up some of their powers which enabled them to credibly commit themselves to their promises not to confiscate the property of their subjects which in turn made the realization of additional gains from cooperation possible. His approach thus flatly contradicts the more conventional one that conceptualizes the emergence of parliament as the consequence of weak autocrats. Here, our focus is not so much on explaining the emergence of institutions that enable actors to make the promises credible but to identify possible equilibria in an essentially institution-free environment.

## 2. Normative versus Positive constitutional economics

Until recently, constitutional economics has almost exclusively been a normative approach. Although the most important representative of the program, JAMES M. BUCHANAN, denies this imbalance between legitimization and explanation<sup>8</sup>, he also admits that *"(t)he purpose of the contractarian exercise is not explanatory in this sense. It is, by contrast, justificatory in that it offers a basis for normative evaluation. Could the observed rules that constrain the activity of ordinary politics have emerged from agreement in constitutional contract? To the extent that this question can be affirmatively answered, we have established a legitimating linkage between the individual and the state"*<sup>9</sup>.

The legitimating link is thus based on unanimous consent. As long as it is factually obtained, this might indeed be sufficient for legitimizing a constitution and subsequent government action. But if the consent is merely hypothetical, i.e. imagined by some social scientist or politician, a legitimating link can only be had if the preferences imputed on the individuals who are to (hypothetically) agree unanimously are uniformly agreed upon. If that is not the case, i.e., if there are quarrels concerning the "correct" imputation of preferences, different observers will be able to legitimate very different constitutions. E.g., if subjects are assumed to be highly risk-averse, they would agree to constitutional rules very different from those they will agree to were they assumed to be risk-neutral. If almost any constitution can be legitimated by somebody or the other, the legitimating link will, obviously, be rather weak.

When constitutional economists try to make a positive argument concerning the emergence of the state, they often draw on social contract theory. They usually model two individuals who are stuck with the Nash-equilibrium of the prisoners' dilemma. Because both could be better off were they able to commit themselves to cooperate, they create the state via social contract enabling them to mutually cooperate.

Suppose that indeed the state is founded by contract as a consequence of the inability of two (or more) potential parties to a private contract to credibly renounce playing the dominant strategy. This amounts to nothing more than the claim that the inability to enter into a mutually beneficial contract can be overcome by entering into yet another contract. Suppose that this last contract was equally the attempt to coordinate behavior in a prisoners' dilemma. Obviously, one would need yet another contract to solve the second-order dilemma and so forth. In other words: the attempt to solve a simple prisoners' dilemma by a social contract is equivalent to entering into an infinite regress.

Whereas the argument just presented focuses on the emergence of constitutions, a similar reproach can be made concerning their stability: suppose the two parties that originally failed to credibly renounce their defect-strategies and consequently also failed to solve the prisoners' dilemma entered into a social contract and founded the state with the intention of establishing an impartial arbitrator and an enforcement agency. The parties to the simple exchange contract then disarmed themselves and paid a fee to the third

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<sup>8</sup>E.g. BUCHANAN (1990, 2).

<sup>9</sup>(1987, 249).

party for its services instead<sup>10</sup>. They endowed the third party with the monopoly of using force. Then what incentives does the third party (the state) have to stick to its role of impartial arbitrator instead of expropriating the two parties which originally founded it?

We thus argue that in order to explain the emergence of the state, the notion of social contract is of little help. Similarly, it is of little help when one is interested in explaining constitutional change. If constitutional rules are assumed to be the most basic layer of rules, they need to be self-enforcing. This does, however, not mean that the notion of contract would become completely superfluous. Indeed, one can conceive of a give and take between a limited number of actors.

It is crucial, however, not to introduce an enforcement agency by fiat but to analyze whether the contracting parties have incentives to stick to the provisions of the contract in the post-constitutional stage. In our model, we therefore only need two players, and not a third one who is created by the original two players. As a consequence of the folk-theorem<sup>11</sup>, proponents of social contract theory could argue that they can also do without a third party as long as the game is repeated often enough. That is why we decided not to focus on repeated games but to ask whether various forms of government could also be explained by analyzing a one-shot game which might, however, be played repeatedly, yet without assuming intertemporal links.

It might be useful to present the central questions of positive constitutional economics since it is still a very young research program<sup>12</sup>: on a very basic level, one can distinguish between constitutional rules as *explanantia* on the one hand and constitutional rules as *explananda* on the other. In the first case, one assumes various constitutional arrangements as given and asks whether they systematically cause variation in variables of interest to the economist. One could, e.g., compare how rule of law constitutions fare with regard to per capita income when compared to autocracies.

Modeling constitutions as *explananda*, one aims at explaining the emergence of constitutions and their change over time. Natural candidates for explanatory variables are the preferences that the relevant actors bring to the bargaining table (the constitutional convention or what have you) as well as the procedures used for reaching a bargaining solution. Here, we are, of course, interested in identifying some variables that determine whether the resulting constitution is one of limited government and would therefore be close to a rule of law constitution, or whether some level of exploitation prevails which would correspond to autocracy.

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<sup>10</sup>KIRSTEIN/SCHMIDTCHEN (1997) provide an analysis of the impact that court costs and imperfect decision-making judges have on contracting parties in a trust game.

<sup>11</sup>See e.g. FUDENBERG/MASKIN (1986).

<sup>12</sup>VOIGT (1997) is a survey.

### 3. The choice between dictatorship and cooperation

#### 3.1 Basic assumptions

We assume society to consist of two groups of individuals, those who have a comparative advantage in using violence and those who have a comparative advantage in producing a private good. Each group consists of a large number of individuals who are treated as homogeneous. We model both groups of individuals as if they were one monolithic actor because we are interested in the analysis of intergroup interaction and not in intragroup interaction. The group commanding the comparative advantage in violence has the capacity to exploit the other group. But the differential in the capacity to produce violence is not unlimited: the costs of overthrowing the exploiting group decrease in exploitation<sup>13</sup>.

The actor commanding an advantage in producing violence has an interest that the other actor works as hard as possible thus producing a high social product of which she would like to secure a share as large as possible. The other actor has an interest in the first actor providing security against external aggression for a price as low as possible.

Drawing on a well-established notion of constitutional economics, we want to distinguish between a constitutional and a post-constitutional stage. In the constitutional stage, the two actors would (implicitly) negotiate on the price of the provision of security. In return, the actor commanding an advantage in producing the private good would promise to choose a certain (high) effort level. We call this agreement a "*social contract*"<sup>14</sup>.

It differs from the usual notion of social contract in at least two aspects:

- no third party which could decide quarrels between the two contracting parties is introduced. The necessity of self-enforceability thus becomes apparent.
- Secondly, the parties negotiating the constitutional agreement are perfectly aware of their identities, thus know whether their comparative advantage is in producing the private good or in producing violence.

The process by which the negotiating parties bring the contract about is not modeled at all. Instead, we focus on the post-constitutional stage, asking what conditions have to be fulfilled in order to make an agreement reached on the constitutional stage sustainable. Coined differently: the scope that the bargaining parties have on the constitutional stage is determined by backward induction focusing on the post-constitutional stage. The **idea** that a simple sheet of paper could bind actors even against their own interest is thus done away with.

Two results are of prime interest: the case in which both actors have incentives to comply with the agreement reached during the first stage. This is called *cooperation*.

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<sup>13</sup>This approach is similar to the one in HENNING/LU (1999), who allow for a probability of a successful revolution that is increasing in the tax rate.

<sup>14</sup>For such an approach to constitutional contracts, see VOIGT (1999, chapter 6).



The first actor sticks to the agreed upon price for the provision of security, the second actor displays a high level of effort. The second result is the one in which the first actor extracts a price surpassing the one agreed upon. We call this equilibrium *exploitation*. We assume that the higher the level of exploitation, the higher the costs that the first actor has to bear in order to execute the chosen level. We further assume that the second actor can always try to depose of the current government (the first actor). The costs of starting such an attempt are assumed to be independent of the level of exploitation chosen by the government. However, the probability of a successful revolution is assumed to be increasing in the production of the private good.

## 3.2 The model

In this paper, we only model the post-constitutional stage. Hence, we take the agreement upon the cooperative effort level and the share of the private good for the more violent group as exogenously given. The two players, called V and W, are assumed to be risk-neutral. V is the group with the comparative advantage in violence. Thus, W has a comparative advantage in producing the private good. The subsequent analysis shows the conditions under which the agreement will be carried out, and under which conditions it is cheap talk and exploitation occurs. In other words, the equilibrium results provide the constraints for the preconstitutional negotiations that (implicitly) take place in each round. This modeling strategy is thus closer to Binmore's approach who conceptualizes the "game of life" as continuously taking place in the state of nature<sup>15</sup>. This refutes the notion of normative constitutional economics according to which the actors are able to erect behavioral constraints in one constitutional stage that will effectively channel their behavior in all of the post-constitutional stages.

We model the post-constitutional stage as a one-shot game, see figure 1. W moves first and decides on the effort level  $e$ , which can either be low or high:  $e \in \{l; h\}$ . The social product is a function of the effort level  $Y = Y_e$  with  $Y_l < Y_h$ . The incremental costs W has to bear if he chooses  $e = h$  are denoted as  $E$ .

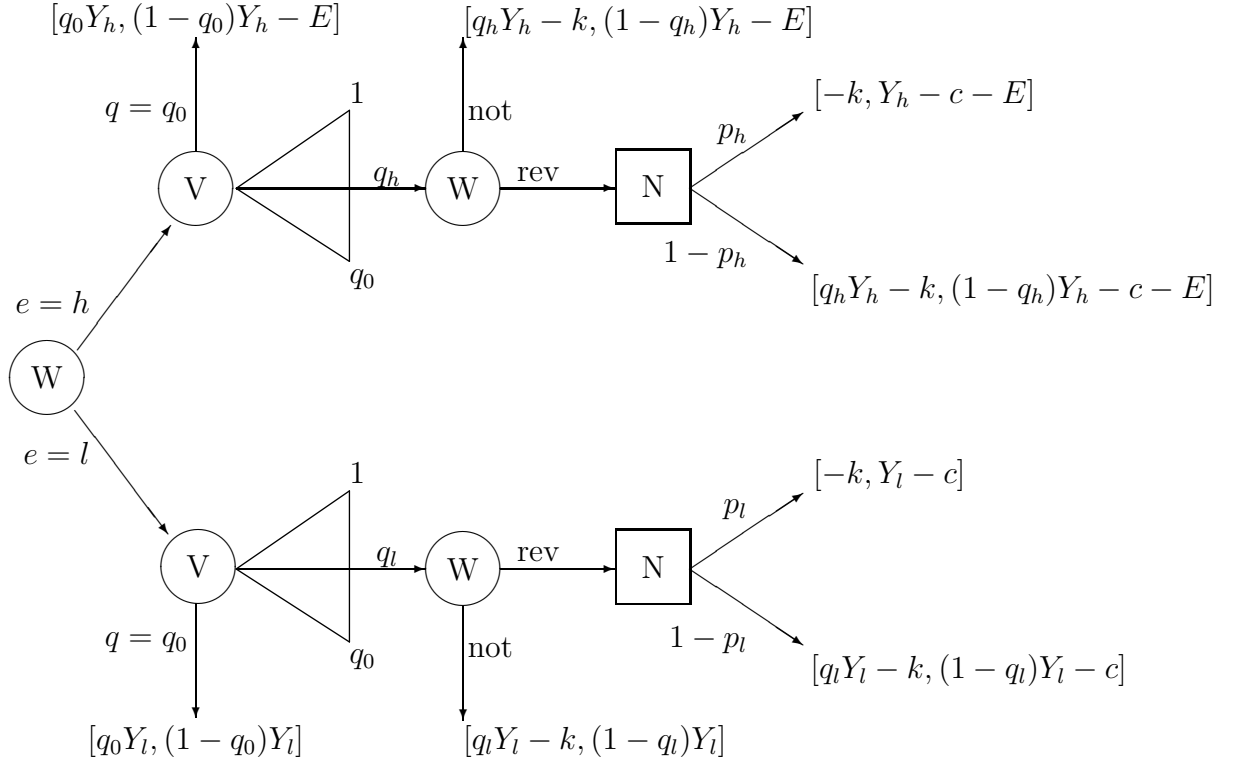
After the input decision of W, his opponent V has to decide whether to demand only the agreed upon share of the private good ( $q_e = q_0$ ) or a higher share ( $q_e > q_0$ ). We take the level of the cooperative share as determined on the constitutional stage and thus as exogenously given in the post-constitutional game under consideration. To demand less than the cooperative share  $q_0$  is a dominated move; hence the choice situation of V at his node can, for the moment, be described as  $q_e = q_0$  versus  $q_e \in ]q_0, 1]$ .  $q_e, e \in \{l; h\}$  denotes the different reactions of V on W's effort choice. If V decides to exploit W at a rate that exceeds  $q_0$ , which is indicated by a triangle in figure 1, he has to bear costs, denoted as  $k > 0$ . These costs are required to produce additional violence - if V chooses to exploit W, he does not only have to protect the external borderlines of the economy, but also has to invest resources into oppression.

If V has chosen exploitation, then W decides whether to attempt a revolution or not. If

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<sup>15</sup>See BINMORE (1994).

Figure 1: The game in extensive form



the revolution is successful, which occurs with probability  $p_e$ , then  $V$  becomes incapable of demanding any of the social product. Whether the revolution is successful or not is modelled as a chance move of "Nature", which is denoted as  $N$ . We assume  $p_h > p_l$ : The more  $W$  has produced, the better are the odds when trying to overthrow  $V$ . In case of failure (with probability  $1 - p_e$ ),  $W$  has to pay the share of the social product demanded by  $V$ , namely  $q_e$ . Successful or not,  $W$  has to bear revolution costs, denoted as  $c$ . For technical reasons, we assume  $q_0 p_e Y_e < c < p_e Y_e$  for all  $e \in \{h; l\}$ .

Figure 1 illustrates some of the driving forces of the model:

- Exploitation at a rate that exceeds the cooperative one ( $q_0$ ) has, at a first glance, a beneficial impact on V's payoff. The same holds for a higher effort of W.
- However, the additional exploitation costs  $k$  can make exploitation harmful for V.
- The same holds for higher effort, which on the one hand makes exploitation more beneficial, yet on the other hand increases the probability of a successful revolution,  $p$ .

Hence, high effort of W is clearly beneficial for V as long as he avoids a revolution - or if V exploits at a rate that overcompensates for the risk of being overthrown. For the moment we simply assume that W does not choose to try to depose of his opponent if V only demands the cooperative share; the analysis below will show that this assumption can also be derived as an endogenous result.

### 3.3 Backward induction

#### 3.3.1 The revolution decision

First, we analyze W's decision whether to attempt a revolution (option rev) or not. Note that, when W has to make this decision, his effort choice and hence the effort costs are already sunk. For the analysis whether it is better to attempt a revolution or not, we thus can neglect the costs of high effort, which are denoted as  $E$ . Moreover, we do not regard which effort W has chosen and present the subsequent analysis without specifying  $e$ .

Given W has chosen his effort level and V has chosen to exploit W (at a rate  $q_e$  that exceeds  $q_0$ ), then W receives

$$(1 - q_e)Y_e$$

if he adapts to the exploitation. The revolution, on the other hand, yields

$$p_e[Y_e - c] + (1 - p_e)[(1 - q_e)Y_e - c]$$

for W. This equals  $p_e Y_e + (1 - q_e)Y_e - p_e(1 - q_e)Y_e - c$  and

$$(1 - q_e)Y_e + p_e q_e Y_e - c$$

Thus, this is W's expected payoff from attempting a revolution, which exceeds his payoff from adapting to exploitation if, and only if, the following condition holds:

$$p_e q_e Y_e > c$$

We call this expression the *revolution condition*. If the revolution condition does not hold, i.e. if  $c \geq p_e q_e Y_e$ , then it is beneficial for W not to attempt the revolution.

Figure 2: The revolution condition

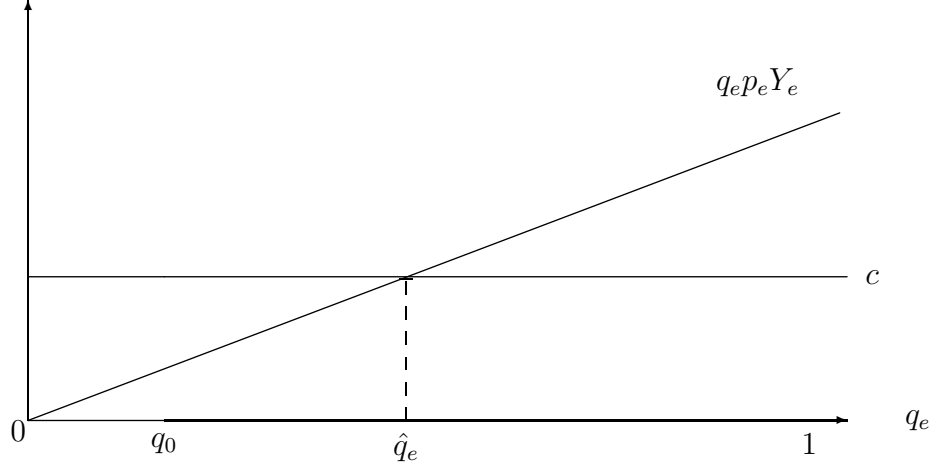


Figure 2 visualizes the revolution condition: The horizontal line represents the revolution costs  $c$ , the diagonal line stands for  $q_e p_e Y_e$ , the expected gain from revolution. The revolution condition allows to derive a threshold value that we denote as  $\hat{q}_e$ , with

$$\hat{q}_e = \frac{c}{p_e Y_e}$$

To the left of this threshold, the revolution condition does not hold, hence (not) is the preferred option. To the right of  $\hat{q}_e$ , revolution is beneficial.

### 3.3.2 The exploitation decision

The analysis above showed that V triggers the subsequent behavior of W when demanding his share of the social product. If he chooses  $q_e < \hat{q}_e$ , then W will not attempt a revolution<sup>16</sup>. Now it is obvious that after  $q_e = q_0$  V does not have to fear a revolution.

If, however, V chooses  $q_e > \hat{q}_e$ , then he faces the lottery of a revolution. His expected payoff then is  $(1 - p_e)q_e Y_h - k$ , because in case of a successful revolution, V receives no share of the social product and still has to bear the additional costs of violence.

V now has to distinguish three cases:

<sup>16</sup>Again, in this section, we can neglect the incremental costs of high effort,  $E$ .

- If he chooses  $q_0$ , his payoff is  $q_0 Y_e$ . This saves him the additional costs of  $k$ .
- If he wants to exploit W, but avoid a revolution, he has to choose  $q_e \in ]q_0, \hat{q}_e]$ . His payoff then is  $q_e Y_e - k$ .
- If he accepts the risk of being overthrown, then he chooses  $q_e \in ]\hat{q}_e, 1]$  and receives  $(1 - p_e)q_e Y_e - k$  as the expected payoff.

Obviously, in all of the three cases, the (expected) payoff of V is monotonically increasing in  $q_e$ . Hence, only three of his options are relevant for V:

- a)  $q_e = q_0$ , i.e. to cooperate.
- b)  $q_e = \hat{q}_e$ , i.e. moderate exploitation that avoids revolution.
- c)  $q_e = 1$ ; i.e. the maximum exploitation, facing the risk to get overthrown in the following revolution.

Given an effort choice  $e$ , the option  $q_0$  is optimal, if the payoff  $q_0 Y_e$  exceeds both  $\hat{q}_e Y_e - k$  (from moderate exploitation) and  $(1 - p_e)Y_e - k$  (from the maximum exploitation with revolution). Moderate exploitation, i.e.  $q_e = \hat{q}_e$ , is optimal if  $\hat{q}_e Y_e - k$  exceeds both the payoff from cooperation ( $q_0 Y_e$ ) and from exploitation with revolution, which is  $(1 - p_e)Y_e - k$ .

Thus, we can derive the following

**Proposition 1:** *Given a social contract  $(q_0, h)$ , an effort choice  $e \in \{h; l\}$ , probabilities of success of a revolution  $p_h > p_l$ , revolution costs  $c$  with  $q_0 p_e Y_e < c < p_e Y_e$ , then the optimal action of V is*

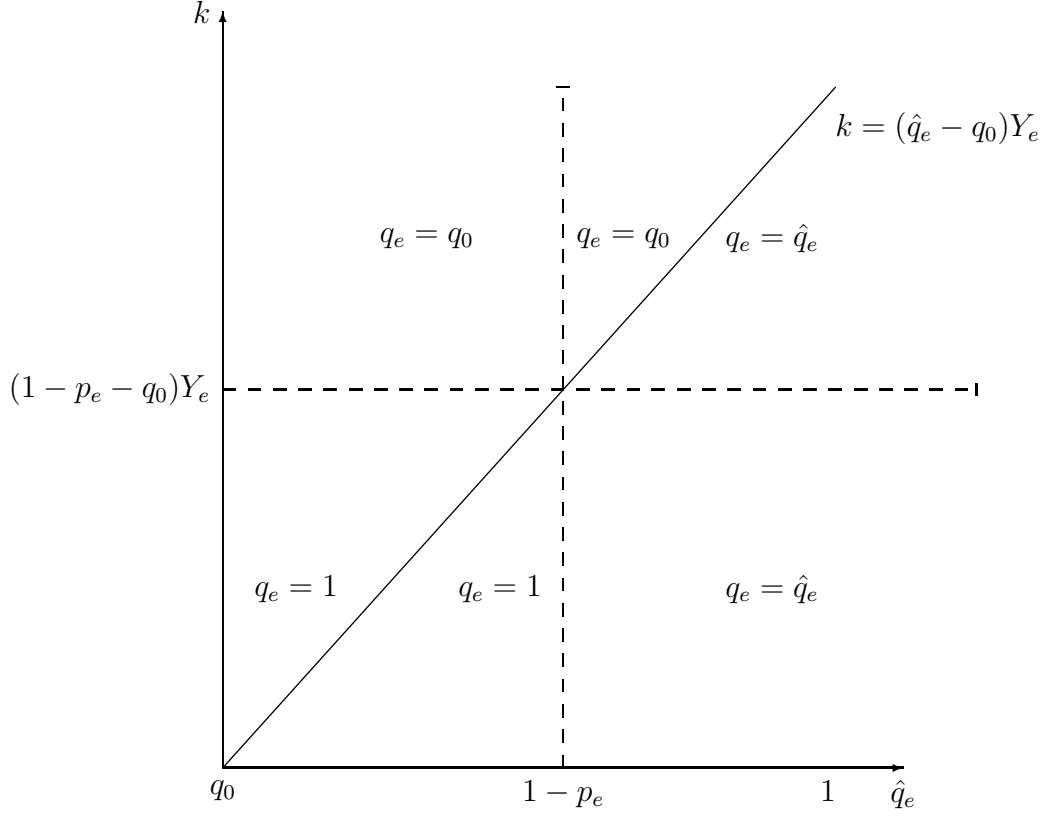
- $q_e = q_0$  if, and only if,  $k > (\hat{q}_e - q_0)Y_e$  and  $k > (1 - p_e - q_0)Y_e$ ;
- $q_e = \hat{q}_e$  if, and only if,  $k < (\hat{q}_e - q_0)Y_e$  and  $\hat{q}_e > (1 - p_e)$ ;
- $q_e = 1$  if, and only if,  $k < (1 - p_e - q_0)Y_e$  and  $\hat{q}_e < (1 - p_e)$ .

This proposition is visualized in figure 3. The graph shows combinations of the parameters  $k$  and  $\hat{q}_e$ , which are exogenous<sup>17</sup>. Figure 3 shows, for all of the possible combinations of  $k$  and  $\hat{q}_e$ , V's optimal reaction  $q_e$  on the effort chosen by W, with  $q_e \in \{q_0; \hat{q}_e; 1\}$ . Above the horizontal dashed line, the condition  $k > (1 - p_e - q_0)Y_e$  is fulfilled. To the right of the vertical dashed line,  $\hat{q}_e > 1 - p_e$  holds. The intersection of the dashed lines as well as the origin of the graph (where  $k = 0$  and  $\hat{q}_e = q_0$  hold) belong to  $k = (\hat{q}_e - q_0)Y_e$ . Hence, above the diagonal line,  $k > (\hat{q}_e - q_0)Y_e$  holds.

The areas above and below the lines in figure 3 represent the parameter combinations that allow to derive V's optimal choice. E.g., in the lower left corner, there are two areas where harsh exploitation ( $q_e = 1$ ) is optimal. To the right, but below the diagonal line,  $q_e = \hat{q}_e$  is the optimal choice. In the upper areas,  $q_e = q_0$  is optimal. Note that the borderlines between these areas are governed by

<sup>17</sup> $\hat{q}_e$  is determined by  $c = q_e p_e Y_e$ .  $p_e$  and  $Y_e$  are exogenous, hence  $\hat{q}_e$  is exogenous, too.

Figure 3: The exploitation conditions



- The effort  $e$  chosen by W,
- the exogenous parameters  $c, Y_e, p_e, k$ , and
- the result of the pre-constitutional agreement  $q_0$ .

With a high  $k$ , only cooperation or moderate exploitation can be optimal for V, whereas under a lower  $k$ , either maximum or moderate exploitation is optimal. Whether moderate exploitation is the best choice depends on  $\hat{q}$ .

### 3.3.3 The effort decision

Our last step of the backward induction analysis is to derive the parameter settings for the optimal effort choice of W. Now the incremental costs of high effort, denoted as  $E$ , become relevant for the analysis. In this paper, we are not interested in a complete equilibrium analysis. Our focus is to derive the conditions for stable cooperation (rule of law) and stable exploitation. Hence, we only derive the conditions for the following two equilibrium paths:

1.  $(e = h, q_h = q_0)$ , i.e., compliance with the social contract.
2.  $(e = l, q_l = \hat{q}_l, \text{not})$ , i.e., stable exploitation.

According to the proposition derived above, it depends on the parameters  $k, c, p_e, Y_e$  and  $q_0$  whether  $q_0, \hat{q}_e$  or 1 is the optimal reaction of V on a particular effort choice. Since W can choose his effort level  $e$  out of  $\{h; l\}$ , there exist nine possible combinations of optimal reactions  $q_h$  and  $q_l$ . For each of these nine cases, a condition can easily be derived under which V prefers the high effort.

For simplification, we limit our view to one out of the nine possible cases, namely the one where  $(q_l = \hat{q}_l, q_h = q_0)$  reflects the optimal behavior of V. The necessary and sufficient condition for this case is

$$k > (1 - p_h - q_0)Y_h \wedge \quad (1)$$

$$k > (\hat{q}_h - q_0)Y_h \wedge \quad (2)$$

$$\hat{q}_l > 1 - p_l \wedge \quad (3)$$

$$k < (\hat{q}_l - q_0)Y_l \quad (4)$$

The first two of these conditions make  $q_0$  the optimal reply on  $e = h$ , whereas conditions (3) and (4) provide that  $q_0$  is optimal in case of  $e = l$ . If the conditions (1) through (4) hold, then W receives  $(1 - q_0)Y_h - E$  if he chooses  $e = h$ . His expected payoff from  $e = l$  is, on the other hand,  $(1 - \hat{q}_e)Y_l$ . Thus, he prefers high effort over low if, and only if,

$$E < (1 - q_0)Y_h - (1 - \hat{q}_l)Y_l \quad (5)$$

This leads to our second proposition:

**Proposition 2:** *Given a social contract  $(q_0, h)$ . If the conditions (1) through (5) hold, the equilibrium path of the game is  $(e = h, q_h = q_0)$ . If, on the other hand, conditions (1) through (4), and not (5) hold, then the equilibrium path is  $(e = l, q_l = \hat{q}_l, \text{not})$ .*

Note that this proposition states sufficient (but not necessary) conditions for the two equilibrium paths that we analyze here. If either of the conditions (1) through (4) were violated, this would have an impact on the payoffs W had to take into consideration when making his effort decision. This decision is governed by condition (5) if conditions (1) through (4) hold.

### 3.4 The impact of parameter changes

In this section, we will systematically analyze the impact of changes of the parameters  $Y_h, Y_l, p_h, p_l, c, k$  and  $E$  on the five conditions that were derived above. In doing so, the social contract parameters  $(h, q_0)$  are of special interest; recall that  $q_0 = c/p_e Y_e$ .

Let us begin with the conditions for  $q_h = q_0$ , namely (1) and (2). The conditions (1) and (2) are more likely to be fulfilled,

- the higher  $k, p_h, Y_h$  and  $q_0$ ,
- and the lower  $c$

If only condition (1) is violated, then it is obvious from figure 3 on page 13 that the optimal answer on  $e = h$  is  $q_h = 1$ . If only (2) does not hold, then the optimal  $q_e$  would be  $\hat{q}_h$ .

Things are less obvious with respect to the conditions (3) and (4). Condition (4) is  $k < (\hat{q}_l - q_0)Y_l$ . Using  $\hat{q}_e = c/p_e Y_e$  leads to  $k < c/p_l - q_0 Y_l$ , which is easier to evaluate. Condition (3) is equivalent to  $c/p_l Y_l > 1 - p_l$ , which can be transformed to  $c > (p_l - p_l^2)Y_l$ . Taking into account that  $p_l \in [0, 1]$ , we can derive that the conditions (3) and (4) are more likely to be fulfilled,

- the higher  $c, Y_l$  and  $q_0$ ,
- and the lower  $p_l$

If only condition (3) is violated, V's optimal answer on  $e = l$  would be  $q_l = 1$ . The violation of condition (4) alone would make  $q_l = q_0$  optimal.

Finally, we analyze condition (5), which is equivalent to  $E < (1 - q_0)Y_h - Y_l + c/p_l$ . This condition is more likely to be fulfilled,

- the higher  $Y_h$  and  $c$ ,
- and the lower  $E, Y_l, p_l$  and  $q_0$ .

With respect to the social contract parameters, these considerations lead to the following results: In order to implement the equilibrium path ( $e = h, q_h = q_0$ ),  $Y_h$  should be as high as possible. It is only the effort condition that keeps the parties from contracting a very high  $h$ . The share of the violent party,  $q_0$ , on the other hand, must not be too high (due to the effort condition), and not too low (due to the exploitation condition).

The difference between  $Y_h$  and  $Y_l$  reflects the productivity of a particular effort input  $E$ . If this productivity is high, and the share to the violent group is somehow medium, then a social contract is likely to be fulfilled. With a relatively high  $Y_l$  (or a low productivity), exploitation would occur - the social contract is only cheap talk.

## 4. Conclusions and outlook

In this paper, we have derived the conditions which determine the various possible equilibria concerning the effort level chosen by W and the exploitation level chosen by



V. Section 3.3 served to derive the equilibrium conditions via backward induction. We were especially interested in two of the possible equilibria, namely the one equivalent to compliance with the social contract and the other one equivalent to stable exploitation. In section 3.4, the results derived in 3.3 were interpreted by asking what effects changes in the parameter settings would bring about.

Concerning the chances of a social contract consisting of a promise of W to exert a high effort level and of V to constrain taxation to a predetermined level to be enforced, it was shown that self-enforceability increases with the payoffs of a given E, i.e. with the productivity that additional effort on the side of W entails. It seems worth noting that this result obtains although we only analyze a one-shot game; it is thus not due to intertemporal threats from W to produce less in the next period in case V does not stick to the fair taxation level.

Similarly, it was shown that chances of self-enforceability increase with high exploitation costs. This result is in accordance with our intuitions: the more costly it is for the group producing violence to take away some of the wealth of the other group, the less attractive this option becomes. Formulated the other way round: the more costly it becomes, the more incentives this group has to stick to the implicitly agreed upon taxation level. Finally, it was shown that chances of self-enforceability increase with low revolution costs. This result is also in accordance with our intuitions: the less costly it is to kick off a revolution, the more careful V will be not to renege upon the social contract implicitly agreed upon with W.

It is, of course, tempting to think of an empirical test of this theory. Rather than carrying it out here, some of the problems to be solved will simply be spelled out. To group empirically observable cases into one of the possible equilibrium paths, one would have to be able to agree on their specific situation. Going even one step further back, one would look at the conditions one through five that were derived in 3.3 for a fair taxation level to materialize. Unequivocally agreeing on when these conditions are fulfilled will hardly ever be possible. One of the central insights of this theory is, however, that high productivity increases the chances of self-enforceability. One test-strategy could therefore consist in estimating the productivity of a given effort-level at different times and different places. This estimate could then be compared with income and taxation levels of those societies. One could proceed similarly with exploitation and revolution costs.

Confronting the insights of the model with empirically observable constitutional agreements, a number of questions remain open. Apparently, it can be observed that constitutional arrangements which - due to the specific parameter settings - do not have a real chance of becoming enforced are still concluded and solemnly presented in public. What are the incentives of the actors to invest time and effort into negotiating these documents if their ability to constrain politicians - and other actors - is rather small?

Deviating from the approach usually chosen in constitutional economics, we decided not to model the constitutional stage explicitly but to identify the parameters which determine whether an agreement reached on the constitutional level is enforceable or not. For

a sequel paper, endogenizing the bargaining process taking place on the constitutional stage rather than assuming the contract as exogenously determined seems a logical step. Closely related is another deviation from conventional constitutional economics that was chosen here: we do away with the concept of a third party that would somehow enforce social contracts but do insist on the self-enforceability of such contracts.

The particular novelty of our approach is that we also deviate from well-established concepts of self-enforceability such as those of TELSER (1980), KLEIN (1985), or AXELROD (1984) which are based on repeated games. That is not to say that repetition will not have any effects - it almost certainly will. It could thus be an issue of a sequel paper to deal with the intertemporal effects that arise as a consequence of repetition. One possible aspect is that W could invest in order to enhance its productivity which could possibly lead to a change in the equilibrium path.

It seems worth noting that the analyses presented in 3.4 are concerned with comparative statics which means that we do not deal with the origin of changes in our exogenous variables. If one is interested in the path from one equilibrium to another, i.e. in a dynamic model, this will certainly become an issue. This is, too, a possible topic of a future paper.

In this paper, there are two homogenous actors V and W. In future papers, two modifications would seem to be worthwhile: (1) to give up the assumption that the groups are perfectly organized. In other words, one could introduce latent interest groups that would have to overcome the free-rider problems inherent in collective action in the first instance. (2) to allow for a larger number of groups who make up a uniform W in our model. This would bring the issue of coalition building among them to the fore.

Obviously, the research program of Positive Constitutional Economics is still in its very beginning. Much theoretical and empirical work remains to be done in order to explain the emergence and the modification of self-enforcing, i.e. *de facto* constitutions.

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